# Foundations of Distributional Semantic Models

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#### **Outline**

- Background and motivation
- Defining the DSMs
  - DSMs in a nutshell
  - Generalized DSMs
- The "linguistic" parameters
  - Corpus pre-processing
  - Defining the context
- 4 The "mathematical" parameters
  - Context weighting
  - Dimensionality reduction
- A taxonomy of DSMs

### **Credits**

This course is based on joint work with Marco Baroni (CiMEC, University of Trento), who prepared some of the slides for a previous course on Distributional Semantics Models.

### Where are word meanings?

- Meanings in the world
  - the meaning of *car* is the <u>set</u> of {cars} in this world (extension), or a function from possible words to the sets of {cars} in these worlds (intension, property, etc.)
    - cf. formal semantics
- Meanings in the head
  - the meaning of car is the concept CAR, as a mental representation of the category of cars
    - cf. cognitive psychology
- Meanings in the text
  - the meaning of car is an abstraction over the linguistic contexts in which the word car is used
    - cf. distributional semantics
  - prima facie, a paradox!

### Representing word meaning

- Word meaning is usually represented in terms of some formal, symbolic structure, either external or internal to the word
  - external structure
    - semantic networks (cf. WordNet, Ontologies, etc.)
  - internal structure
    - feature (property, attribute) lists
    - frames (cf. FrameNet)
    - recursive feature structures (cf. Generative Lexicon)
    - predicate structures (cf. DRT, etc.)
- The semantic properties of a word are derived from the formal structure of its representation
  - e.g. inferences, semantic similarity, etc.

### Formal representations of meaning

Some problems (often) left out of the picture

- How to select the right meaning of a word in context?
  - bat → bat<sub>1</sub> (type of mammal); bat<sub>2</sub> (type of artifact)
  - school → school<sub>1</sub> (group of fish); school<sub>2</sub> (location);
     school<sub>3</sub> (institution); school<sub>4</sub> (time), school<sub>5</sub> (group of people) etc.
- How does context affect the meaning of a word?
  - clever politician vs. clever tycoon
  - red hair vs. red wine
- How are meanings acquired?
  - word meaning learning
- How do meanings change?
  - e.g Late Old English docga 'a (specific) powerful breed of dog' > dog 'any member of the species Canis familiaris' (Sagi et al. 2009)

#### Key issue

The relationship between word meaning and word usage in contexts

### Formal representations of meaning

Major assets

- Modelling how word meanings can be composed to build the meaning of a sentence (cf. compositionality)
  - $John \rightarrow john$
  - chases  $\rightarrow \lambda x \lambda y$ .[chase(x, y)]
  - $a \rightarrow \lambda P \lambda Q . \exists x [P(x) \land Q(x)]$
  - $bat \rightarrow \lambda x.[bat(x)]$
  - John chases a bat  $\rightarrow \exists x [\mathbf{bat}(x) \land \mathbf{chase}(\mathbf{john}, x)]$
- Modelling fine-grained lexical inferences
  - John chases a bat ⇒ John chases an animal
  - $kill \rightarrow \lambda x \lambda y.[kill(x,y)] \Leftrightarrow \lambda x \lambda y.$  [CAUSE(x, BECOME(DEAD(y)))]

### In the beginning was the context...

### The Distributional Hypothesis (DH)

- At least certain aspects of the meaning of lexical expressions depend on their distributional properties in the linguistic contexts
- The degree of semantic similarity between two linguistic expressions A and B is a function of the similarity of the linguistic contexts in which A and B can appear

### The DH in linguistics

#### Structuralist linguistics

"If we consider words or morphemes A and B to be more different in meaning than A and C, then we will often find that the distributions of A and B are more different than the distributions of A and C. In other words, difference in meaning correlates with difference of distribution" (Z. Harris. "Distributional Structure", Word, X/2-3, 1954)

#### Corpus linguistics

"You shall know a word by the company it keeps" (J. R. Firth, Selected Papers, 1957)

### Contextual representations

- The definition of contextual representation is consistent with an extended notion of contexts of use of a word, including non-linguistic aspects
  - e.g. aspects of the communicative settings
- De facto, context is equated with linguistic context
  - practical reason it is easy to collect linguistic contexts (from corpora) and to process them
  - theoretical reason it is possible to investigate the role of linguistic distributions in shaping word meaning

### The DH in psychology

#### Contextual representation (Miller & Charles 1991)

- The cognitive representation of a word is some abstraction or generalization derived from the contexts that have been encountered
- A word's contextual representation is an abstract cognitive structure that accumulates from encounters with the word in various (linguistic) contexts
  - a contextual representation is not itself a context, but characterizes a set of contexts

### From linguistic distributions to meaning

Landau & Gleitman (1985); McDonald & Ramscar (2001); Fisher & Gleitman (2002)

- The linguistic structures in which words appear are important clues about their meaning
  - The man gorped Mary the book
  - John sebbed that he was unhappy
  - He filled the wampimuk with the substance, passed it around and we all drunk some
  - We found a little, hairy wampimuk sleeping behind the tree
- We learn the meaning of many terms simply from language (often before having any experience with the corresponding entitities)
  - cf. idiosyncrasy, apotropaic, justice, synchrotron, etc.

### Weak and Strong DH

Lenci (2008)

#### Weak DH

A quantitative method for semantic analysis and lexical resource induction

- word meaning (whatever this might be) is reflected in linguistic distributions
- by inspecting a relevant number of distributional contexts, we may identify those aspects of meaning that are shared by words that have similar contextual distributions

applications E-language modeling, lexicography, NLP

 word sense disambiguation, ontology and thesauri learning, relation extraction, question answering, etc.

### Distributional Semantic Models (DSMs)

- Computational models that build contextual semantic representations from corpus data
- DSMs are models for semantic representations...
  - the semantic content is represented by a vector
  - ... and for the way semantic representations are built
    - · vectors are obtained through the statistical analysis of the linguistic contexts of a word
- Alternative names for DSMs.
  - corpus-based semantics
  - statistical semantics
  - geometrical models of meaning
  - vector semantics
  - word (semantic) space models

### Weak and Strong DH

Lenci (2008)

#### Strong DH

A cognitive hypothesis about the form and origin of semantic representations

- word distributions in context have a specific causal role in the formation of the semantic representation for that word
- the distributional properties of words in linguistic contexts explains human semantic behavior (e.g. judgment of semantic similarity)

applications I-language modeling, concept modeling

 semantic priming, word learning, semantic deficits, etc.

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- A taxonomy of DSMs

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### Collecting context counts for target word "dog"

contexts = nouns and verbs in the same sentence

The dog barked in the park.
The owner of the dog put him on the leash since he barked.

#### DSMs in a nutshell

#### Distributional vectors

- count how many times each target word occurs in a certain context
- build vectors out of (a function of) these context occurrence counts
- similar words will have similar vectors

#### Caveat

- similar vectors represent words that have similar distributions in contexts
- DH is the "bridging assumption" that turns distributional similarity into semantic similarity

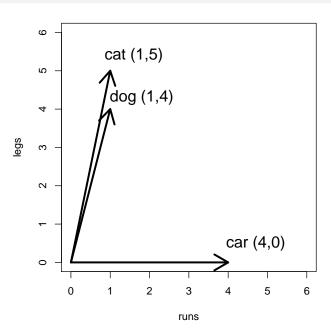
### Contextual representations as distributional vectors

distributional matrix = targets X contexts

#### contexts

		leash	walk	run	owner	leg	bark
targets	dog	3	5	1	5	4	2
	cat	0	3	3	1	5	0
	lion	0	3	2	0	1	0
	light	0	0	0	0	0	0
	bark	1	0	0	2	1	0
	car	0	0	4	3	0	0

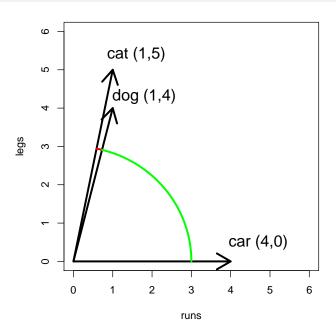
## Semantic space



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### Semantic similarity as angle between vectors



### A general definition of DSMs

- DSMs are tuples < *T*, *C*, *R*, *W*, *M*, *d*, *S* >
  - T target elements, i.e. the words for which the DSM provides a contextual representation
  - C contexts, with which T cooccur
  - R relation, between T and the contexts C
  - W context weighting scheme
  - M distributional matrix,  $T \times C$
  - d dimensionality reduction function,  $d: M \rightarrow M'$
  - S distance measure, between the vectors in M'

### Building a DSM step-by-step

#### The "linguistic" steps

Pre-process a corpus (to define targets and contexts)



Select the targets and the contexts

#### The "mathematical" steps

Count the target-context co-occurrences



Weight the contexts (optional, but recommended)



Build the distributional matrix



Reduce the matrix dimensions (optional)



Compute the vector distances on the (reduced) matrix

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### The DSM parameter space

- Each step determines a wide number of parameters to be fixed
  - which type of context?
  - which weighting scheme?
  - which similarity measure?
  - etc.
- A specific parameter setting determines a particular type of DSM (e.g. LSA, HAL, etc.)

#### Caveat

Parameter setting dramatically affects the resulting semantic space

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### Corpus pre-processing

- Minimally, corpus must be tokenized
- Types of pre-processing
  - POS tagging
  - lemmatization
  - dependency parsing
- Trade-off between deeper linguistic analysis and
  - need for language-specific resources
  - possible errors introduced at each stage of the analysis
  - more parameters to tune
- Corpus processing strategy affects the target and context selection

### Same corpus (Repubblica), different pre-processing

Nearest neighbours of arrivare "arrive"

#### tokenized corpus

- giungere
- raggiungere
- arrivi
- raggiungimento
- raggiunto
- trovare
- raggiunge
- arrivasse
- arriverà
- concludere

### lemmatized corpus

- giungere
- aspettare
- attendere
- arrivo-nn
- ricevere
- accontentare
- approdare
- pervenire
- venire
- piombare

### Same corpus (BNC), different pre-processing

Nearest neighbours of walk

#### tokenized corpus

- stroll
- walking
- walked
- go
- path
- drive
- ride
- wander
- sprinted
- sauntered

#### lemmatized corpus

- hurry
- stroll
- stride
- trudge
- amble
- wander
- walk-nn
- walking
- retrace
- scuttle

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#### Documents as contexts

C = documents, passages, etc.

R = target occurs in C

< doc id = "1" > The silhouette of the sun beyond a wide-open bay on the lake< /doc> < doc id =" 2" > The sun still glitters although evening has arrived in Kuhmo. The sun light is really nice < /doc > < doc id =" 3" > It's midsummer; the living room has its instruments and other objects in each of its corners. < /doc >

- Parameters type and size of documents
  - full document
  - paragraph
  - passage

#### Words as contexts

C = some subset of the lexical words

R = some syntagmatic link connecting the target to C

- C is typically chosen as the n most frequent words (except for a number of stop words)
- Other a priori criteria are possible
  - e.g. nouns as contexts for verbs, particular adverbs as contexts for verbs, verbs of communication as contexts for nouns, etc.
- Types of syntagmatic relations
  - linear
    - word window
    - linguistic unit (e.g. clause, sentence, paragraph etc.)
  - syntactic dependency
  - lexico-syntactic pattern

#### Documents as contexts

distributional matrix = term X document

cf. Latent Semantic Analysis (LSA)

#### documents

	doc <sub>1</sub>	doc <sub>2</sub>	doc
sun	1	2	0
instrument	0	0	1
corner	1	0	1

#### Words as contexts

Linear relations - word window

R = T occurs within a window of n words from C

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners.

#### Parameters

- window size
- window shape
  - rectangular all words in the window have the same weight (cf. Infomap NLP)
  - triangular words closer to the target have a higher weight (cf. HAL)
- window boundary

### Same corpus (BNC), different window sizes

Nearest neighbours of dog

#### 2-word window

- cat
- horse
- fox
- pet
- rabbit
- pig
- animal
- mongrel
- sheep
- pigeon

#### 30-word window

- kennel
- puppy
- pet
- bitch
- terrier

- cat

- rottweiler
- canine
- to bark
- Alsatian

### Words as contexts

Dependency-based relations

R = T is linked to C by a syntactic dependency (e.g. subject, modifier, etc.)

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners.

#### **Parameters**

- types of syntactic dependency (cf. DV; Padó & Lapata 2007)
- type of dependency path
  - direct dependencies
  - direct + indirect dependencies
- length of dependency path

#### Words as contexts

Linear relations - linguistic unit

R = T is in the same linguistic unit as C

The silhouette of the sun beyond a wide-open bay on the lake; the sun still glitters although evening has arrived in Kuhmo. It's midsummer; the living room has its instruments and other objects in each of its corners.

Parameters

- type of linguistic unit
  - sentence
  - paragraph
  - turn in a conversation

#### Words as contexts

Pattern-based relations

R = T is linked to C by a lexico-syntactic pattern (cf. Hearst 1992, Pantel & Pennacchiotti 2008, etc.)

In Provence, Van Gogh painted with bright colors such as red and yellow. These colors produce incredible effects on anybody looking at his paintings.

Parameters

- type of lexical patterns
  - lots of research to identify semantically interesting patterns (cf. Almuhareb & Poesio 2004; Veale & Hao 2008, etc.)

### Contexts and syntagmatic relations

Context-filtering by syntagmatic relations

window-based (Rapp 2003, Infomap NLP)

- Syntagmatic relations as context-filtering functions
  - only those words that are linked to the targets by a certain relation are selected
- Syntagmatic relations as context-typing functions
  - relations define types of contexts

A dog bites a man. A man bites a dog. A dog bites a man.

### Context-typying by syntagmatic relations

window-based (HAL)

Words to the left and to the right of the target are treated as different types of contexts

A dog bites a man. A man bites a dog. A dog bites a man.

### Context-filtering by syntagmatic relations

dependency-based (Padó & Lapata)

A dog bites a man. A man bites a dog. A dog bites a man.

### Context-typing by syntagmatic relations

dependency-based (Grefenstette 1994, Lin 1998, Curran & Moens 2002, Baroni & Lenci 2009)

Words linked to the target with different syntactic dependencies are treated as different types of contexts

A dog bites a man. A man bites a dog. A dog bites a man.

	bite-subj	bite-obj
dog	2	1
man	1	2

### A taxonomy of contexts

- Contexts as documents
  - subtype of contexts depend on the document size and type
    - full documents, paragraphs, passages, etc.
- Contexts as words
  - syntagmatic relation as filters
    - linear relation word window, linguistic unit
    - syntactic dependency
    - lexico-syntactic pattern-based
  - syntagmatic relation as types
    - linear relation word window, linguistic unit
    - syntactic dependency
    - lexico-syntactic pattern-based

### Filters vs. types

- With filters, data less sparse (*man kills* and *kills man* both map to a *kill* dimension of the *man* vector)
- With types
  - more sensitivity to semantic distinctions (*kill-subj* and *kill-obj* are rather different things!)
  - syntagmatic relations provide a form of "typing" of space dimensions (the "subject" dimensions, the "for" dimensions, etc.)
  - important to account for word-order and compositionality in DSMs (cf. Friday class)

### Main opposition in DSMs

- Contexts as documents
  - two words are distributionally similar to the extent that they occur in the same documents
- Contexts as words
  - two words are distributionally similar to the extent that they cooccur with the same words
- Sahlgren (2006) reports very little overlap between these DSM types
  - NB: "contexts as documents" = "syntagmatic spaces" and "contexts as words" = "paradigmatic spaces" in Sahlgren's terminology

### General trends in "context engineering"

- In computational linguistics, tendency towards using more linguistically aware contexts, but "jury is still out" on their utility (Sahlgren in press)
  - this is at least in part task-specific
- In cognitive science trend towards broader document-/text-based definition of contexts
  - focus on topic detection, gist extraction, text coherence assessment
  - Latent Semantic Analysis, Topic Models (Griffiths et al 2007)

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### Context weighting

- From raw counts to log-frequency, to smooth high frequency differences
- Association measures (Evert 2005) are used to give more weight to contexts that are more significantly associated with a target word
  - the less frequent the target word and (more importantly) the context element are, the higher the weight given to their observed co-occurrence count should be (because their expected chance co-occurrence frequency is low)
    - co-occurrence with frequent context element *time* is less informative than co-occurrence with rarer *tail*
  - different measures e.g., Mutual Information, Log-Likelihood Ratio – differ with respect to how they balance raw and expectation-adjusted co-occurrence frequencies
- Information Retrieval weighting schemes
  - word entropy, tf-idf, etc.

### Context weighting

The basic intuition

word1	word2	freq 1 2	freq 1	freq 2
dog	small	855	33,338	490,580
dog	domesticated	29	33,338	918

### Other weighting methods

MI is sometimes criticized (e.g., Manning & Schütze 1999) because it only takes relative frequency into account, and thus overestimates the weight of rare events/dimensions:

word1	word2	freq 1 2	freq 2	MI core
dog	domesticated	29	918	0.03159
dog	sgjkj	1	1	1

#### **Mutual Information**

Church & Hanks (1990)

$$MI(w_1, w_2) = \log_2 rac{P_{\text{corpus}}(w_1, w_2)}{P_{\text{ind}}(w_1, w_2)}$$
 $MI(w_1, w_2) = \log_2 rac{P_{\text{corpus}}(w_1, w_2)}{P_{\text{corpus}}(w_1)P_{\text{corpus}}(w_2)}$ 
 $P(w_1, w_2) = rac{fq(w_1, w_2)}{N}$ 
 $P(w) = rac{fq(w)}{N}$ 

### Other weighting methods

- A popular alternative is the Log-Likelihood Ratio (Dunning 1993)
- "Core" of main term of log-likelihood ratio:

$$fq(w_1, w_2) \times MI(w_1, w_2)$$

 this term alone is also called Local Mutual Information (Evert 2008)

word1	word2	freq 1 2	MI	LLR core
dog	small	855	3.96	3382.87
dog	domesticated	29	6.85	198.76
dog	sgjkj	1	10.31	10.31

For mode details on association measures:

http://www.collocations.de

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### Singular Value Decomposition

- General technique from Linear Algebra (essentially, the same as Principal Component Analysis, PCA)
- given a matrix (e.g., a word-by-context matrix) of  $m \times n$  dimensionality, construct a  $m \times k$  matrix, where k << n (and k < m)
  - e.g., from a 20,000 words by 10,000 contexts matrix to a 20,000 words by 300 "latent dimensions" matrix
  - k is typically an arbitrary choice
- From linear algebra, we know that and how we can find the reduced  $m \times k$  matrix with orthogonal dimensions/columns that preserves most of the variance in the original matrix

More details to come from Stefan!!

### Dimensionality reduction

- Reduce the target-word-by-context matrix to a lower dimensionality matrix
- Two main reasons:
  - smoothing capture "latent dimensions" that generalize over sparser surface dimensions (cf. SVD)
  - efficiency/space sometimes the matrix is so large that you don't even want to construct it explicitly (cf. Random Indexing)

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### The DSM parameter space

- Linguistic parameters
  - pre-processing and linguistic annotation raw text, stemming, POS tagging and lemmatisation, (dependency) parsing, semantically relevant patterns
  - choice of context document, sentence, window, dependency relations, etc.
- Mathematical parameters
  - context weighting log-frequency, association scores, entropy, etc.
  - measuring distance cosine similarity, Euclidean, Manhattan, Minkowski (p-norm)
  - dimensionality reduction feature selection, SVD projection (PCA), random indexing
- A careful understanding of the effects of these parameters on the semantic properties identified by DMSs is still lacking
  - cf. Bullinaria & Levy 2007, Bullinaria 2008 for a systematic exploration of some of these parameters

#### Some instances of DSMs

### Infomap NLP (Widdows 2004)

context rectangular window-based

matrix word X word

W frequency

d SVD

S cosine

### Random Indexing (Karlgren & Salhgren 2001)

context rectangular window-based

matrix word X word

W various

d RI

S various

#### Some instances of DSMs

#### Latent Semantic Analysis (Landauer & Dumais 1996)

context documents

matrix word X document

W log term frequency and term entropy in the corpus

d SVD

S cosine

# Hyperspace Analogue to Language (Lund & Burgess 1996)

context triangular window-based with position as context-typing function

matrix word X word

W frequency

d dimensions with the highest variance

S Minkowski metric

#### Some instances of DSMs

#### Dependency Vectors (Padó & Lapata 2007)

context dependency-based, with dependency as contextfiltering functions

matrix word X word

W log-likelihood ratio

d none

S information theoretic similarity measure in Lin (1998)

#### Distributional Memory (Baroni & Lenci 2009)

context dependency-based, with dependencies as context-typing functions

matrix various

W local MI

d none

S cosine

### Three properties of representations in DSMs

- Distributed meaning is not represented in terms of some conceptual or formal symbol, but in terms of a n-dimensional vector
  - vector dimensions are (typically) semantically empty
  - semantic properties derive from global vector comparison (e.g. by measuring their distance in space)
- Distributional word meaning derives from its distributional history, as recorded in the word vector
- Quantitative and gradual words differ not only for the contexts in which they appear, but also for the salience of these contexts (cf. context weighting scheme)

#### Homework

- Using the online interface WebInfomap, find the nearest neighbors of the following words
  - car
  - president
  - destruction
  - kill
  - build
  - speak
  - red
  - clever
- Analyze the types of neighbors you get with each words, focussing on:
  - the neighbor POS
  - the type of semantic relation with the target (e.g. synonymy, hyperonymy, anonymy, others)
  - differences wrt the window size

#### DSMs and their relatives

- The distributed and quantitative nature of DSM representations make them similar to representations in connectionist models (cf. Rogers et al. 2004)
  - in neural networks, representations are distributed vectors, but not necessarily distributional
    - vectors dimension may encode different type of information, e.g. sensory-motor
- DSM-like representations can also built with neural networks
  - Borovsky & Elman (2006) use Simple Recurrent Networks to model word semantic learning from the distributional analysis of linguistic input (using child-directed speech as a corpus)

### Tomorrow's program

#### Stefan

Matrix algebra and vector spaces